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CURRICULUM GUIDE

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# TAXONOMY OF MATHEMATICS IX OBJECTIVES

## WITH ILLUSTRATIVE TEST ITEMS: A SUMMARY DESCRIPTION

(FIRST EDITION)



Prepared At The Direction Of  
THE JUNIOR HIGH SCHOOL EXAMINATIONS BOARD  
DEPARTMENT OF EDUCATION

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**TAXONOMY OF MATHEMATICS IX OBJECTIVES**  
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## ACKNOWLEDGEMENT

This booklet was prepared by a special committee appointed by the Junior High School Examinations Board. The committee consisted of the following members:

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## FOREWARD

This pamphlet has been prepared in an attempt to adapt the principles of Bloom's Taxonomy of Educational Objectives: Handbook I<sup>1</sup> to the objectives of the existing program of Grade Nine Mathematics as outlined in the Program of Studies for Junior High Schools of Alberta, 1963.

This edition includes objectives used in the instruction and evaluation of modern mathematics in accordance with the program based on the two text series Exploring Modern Mathematics and Seeing Through Mathematics as followed in junior high schools of Alberta.

It is expected that the Grade Nine Mathematics Examinations Committee, as a separate and distinct committee, will incorporate the principles of this taxonomy as is deemed necessary in developing the Grade Nine Departmental Examinations.

To simplify the principles underlying the taxonomy and their adaptation to the objectives of the mathematics program, this pamphlet has been prepared in three parts as shown:

	Title	Purpose
Part I	Interrelationship of Objectives, Teaching and Evaluation	To outline the three levels of objectives in the Mathematics program and to relate them to teaching and evaluation
Part II	Outline of Bloom's Taxonomy	To provide an explanation of Bloom's classification system
Part III	Specific Operational Objectives and Illustrative Test Items	To provide teachers with a more detailed explanation of taxonomic categories and to provide sample, close-range operational objectives along with accompanying test items based on the present course of studies

A blueprint of the Grade IX Mathematics Examination has not been incorporated in this pamphlet. Such a plan is subject to annual revision by the Committee of Examiners and therefore not the concern of the committee constructing this taxonomy. It is hoped that a test blueprint will be prepared and distributed by the Junior High School Examinations Board.

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<sup>1</sup>Bloom, B.S. (ed.), Taxonomy of Educational Objectives The Classification of Educational Goals, Handbook I: Cognitive Domain, New York: David McKay Company, Inc., 1956

PART I  
INTERRELATIONSHIP OF OBJECTIVES, TEACHING  
AND EVALUATION

The process of teaching consists primarily of three tasks. The first is formulating the general, specific and operational objectives of a course; the second is teaching towards these objectives in the most effective and efficient manner; the third is evaluating the progress of the pupils towards these objectives. Teachers who have a clear conception of the objectives of a course or a course unit will be more effective in the classroom and will be more accurate in the evaluation of pupil achievement.

The current curriculum guides\* state the objectives of the junior high school program. A summary of these objectives follows in the next section.

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\* Curriculum Guide for Grade IX Mathematics, 1967



## OBJECTIVES OF GRADE IX MATHEMATICS

Unlike most of the other areas of knowledge, mathematics is entirely an invention of man. It was developed as a means of describing and studying the phenomena he observed in the world around him. As such, mathematics is a basic subject. In order to understand the world around us it is necessary to understand the basic concepts of mathematics and to develop skill with its fundamental processes.

In a program of mathematics instruction much of the emphasis must be placed on the learning of basic concepts, fundamental processes, methods of manipulation and the properties of number systems. However, all of this will be of no avail unless the student has developed an understanding of the power of mathematics when applied in such fields as science, engineering and social sciences such as economics. Without these understandings and skills it is impossible for a student to take an effective part in the world today, or to proceed far in education beyond high school.

The general objective of Grade IX Mathematics instruction is to enable the student to acquire skills in the fundamental processes and to begin to develop understandings of basic concepts and appreciations of the contribution of mathematics to the other sciences. To achieve this general objective the mathematics program has been subdivided into a series of more specific objectives. These objectives are:

1. To develop an understanding of mathematical concepts and an appreciation of mathematical structure.
2. To develop skill in the use of fundamental processes.
3. To develop systematic methods of analyzing problems and presenting their solutions.
4. To develop habits of precise thought and expression.
5. To develop an understanding of the significance and application of mathematics in the modern world.

The preceding five objectives are still not specific enough to be of much value in the day-to-day instruction of students. They must be subdivided further into operational objectives. The major responsibility for developing the operational objectives of day-to-day instruction must rest upon the classroom teacher. Bloom's Taxonomy of Educational Objectives in the Cognitive Domain provides a pattern which the teacher can use in making this analysis. It is impossible to make an exhaustive analysis of the operational objectives



of Grade IX Mathematics within the scope of this document. Thus, the remainder of this booklet illustrates how Bloom's taxonomic system may be applied to sample concepts in the major areas of Mathematics IX. Teachers are urged to carry this analysis further in developing the operational objectives for their own teaching.

## PART II

## OUTLINE OF BLOOM'S TAXONOMY IN RELATION TO GRADE IX MATHEMATICS

As used in the context of Grade IX Mathematics the general objectives are statements of the basic aims of instruction. The specific objectives are sub-divisions of the general objectives that provide some detail as to how the general objectives are to be attained. The operational objectives are still more specific. They act as a practical guide for the day-to-day instructional activities which are carried out in the classroom. They are statements of the knowledge, understandings, manipulative and reasoning skills which students should acquire. As such they are the most appropriate objectives to which teaching activities and test items for evaluation can be directed. Because of their importance to both instruction and evaluation, close-range operational objectives along with corresponding test items receive primary emphasis in this taxonomy.

Bloom and his associates developed a system for classifying educational objectives in terms of the level of pupil thinking involved. The system serves various purposes. One purpose is to provide clear definitions of terms used to describe thought processes. Words such as "understanding", "analysis", and "evaluation" are defined in terms of student behavior, and examples of operational objectives and test items directed towards these thought processes are provided. A second purpose of the taxonomy is to establish a hierarchy of thought processes, ranging from simple recall of meanings of terms to evaluation of a complete work. The system is intended to be exhaustive; that is, every objective and every test item in the cognitive domain, the field of gaining and using knowledge, can be fitted into one of the categories of the classification system. Another purpose of the taxonomy is to assist teachers in constructing tests by relating test items to course objectives.

There are two major divisions of the cognitive domain -- (1) Knowledge and (2) Intellectual Skills and Abilities. The latter is subdivided into the categories of Comprehension, Application, Analysis, Synthesis and Evaluation. Comprehension serves as a transitional area of understanding between Knowledge and the remaining categories.

Application, Analysis, Synthesis, and Evaluation may be classed as higher mental processes in which the student must not only know and understand a concept but must also be able to use it in new and meaningful ways.

The table below shows the divisions and subdivisions of Bloom's Taxonomy. More elaborate explanations and further subdivisions of these categories, along with illustrative objectives and test items follow in Part III of this taxonomy.

### Knowledge

#### 1.00 Knowledge

- 1.10 Knowledge of Specifics
  - 1.11 Knowledge of terminology
  - 1.12 Knowledge of specific facts
- 1.20 Knowledge of Ways and Means of Dealing with Specifics
  - 1.21 Knowledge of conventions
  - 1.22 Knowledge of trends and sequences
  - 1.23 Knowledge of classifications and categories
  - 1.24 Knowledge of criteria
  - 1.25 Knowledge of methodology
- 1.30 Knowledge of the Universals and Abstractions in a Field
  - 1.31 Knowledge of principles and generalizations
  - 1.32 Knowledge of theories and structures

### Intellectual Skills and Abilities

#### 2.00 Comprehension

- 2.10 Translation
- 2.20 Interpretation
- 2.30 Extrapolation

#### 3.00 Application

#### 4.00 Analysis

- 4.10 Analysis of Elements
- 4.20 Analysis of Relationships
- 4.30 Analysis of Organizational Principles

#### 5.00 Synthesis

- 5.10 Production of a Unique Communication
- 5.20 Production of a Plan or a Proposed Set of Operations
- 5.30 Derivation of a Set of Abstract Relations

#### 6.00 Evaluation

- 6.10 Judgement in Terms of Internal Evidence
- 6.20 Judgement in Terms of External Criteria

The Grade IX Mathematics program is divided into major areas of mathematical study. They are as follows:

1. Plane and Space Geometry
2. Mathematical Systems
3. Mathematical Logic
4. Formulas and Variation
5. Algebraic Expressions
  - a. linearity; equality and inequality
  - b. algebraic expressions

These major areas form the basis for the organization of this taxonomy of objectives for Grade IX Mathematics. The area of Plane and Space Geometry is the one in which the most detailed and complete application of Bloom's Taxonomy can be made. Therefore, it has been selected for detailed analysis and illustration of typical test questions. The other areas are treated in much less detail and with fewer examples of typical test items. It is recommended that teachers complete the detail of the taxonomy for each of the other areas.

## VALUE OF THIS "TAXONOMY" TO TEACHERS

This pamphlet presents an adaptation of Bloom's 'taxonomy' to Grade IX Mathematics; that is, the objectives of Grade IX Mathematics are broken down and classified according to Bloom's system. Teachers who familiarize themselves with this taxonomy should gain a broadened perspective of what may be accomplished in teaching and how achievement may be evaluated.

A teacher may use this adaptation to assist in evaluating the effectiveness of his own teaching. If he classifies his operational objectives he can readily determine what levels of thinking he expects the students to master; if he classifies his test items, he can determine the degree to which the students have reached the objectives.

## Problems of Classification

Teachers should be cautioned that it is not always easy to assign either educational objectives or test items to a distinct and clear-cut category. This fact, however, does not detract from the value of attempting accurate classification. At the outset it is important to stress that the classification of any particular operational objective or of any particular test item is NOT determined by the mental activities used by the teacher in arriving at a desired goal or the answer to a problem. What is considered in all cases is the thought process that the student must utilize.

Usually, well defined operational objectives can be classified quite consistently. However, the classification of test items used for evaluating these objectives is more difficult. When the level of thinking required by a question is being judged, two additional factors must be considered:

(1) the students' previous knowledge and experience related to the question, and (2) instructional procedures used in presenting the materials.

Because these factors may vary even among students within the same classroom, classification judgements must be made on the basis of what the individual teacher considers to be the 'average' student. That is, the teacher must judge what thought processes will be used by most pupils in their responses to an item. Normally this will result in a close enough approximation to all students of a particular class or grade. However, it is well to keep in mind that any particular test item may demand a different level of thought for one student than for another, and that teacher opinion about its classification may vary. Exposure to enriched or different types

of experience or instruction may enable one student to answer a question on the basis of memory alone since to him the task is already familiar, while another student may have to call upon skills and abilities of a higher nature to answer the same question because his experience has not previously included the task. It should also be kept in mind that for classification purposes, objectives as well as test items are to be assigned to the category representing the highest level of thinking involved since it is assumed that processes involved at lower levels are also included. For instance, in order to apply, a student must understand, and to understand he must remember.

Teachers might assume that test items at lower levels (for example, Knowledge and Comprehension) should be emphasized when testing less able students, and that items at higher levels (such as Application and Synthesis) should be emphasized when testing more able students. However, each of Bloom's categories can be represented by either simple or complex items applicable to almost any level of ability. Teachers are encouraged to implement a wide range of objectives in both teaching and testing.

Teachers are encouraged to refer to other sources of ideas related to the understanding and application of principles underlying the present taxonomy. These include publications such as the "Summary Descriptions" and "Taxonomies" (1, 2, 3, 4) (with respective statistical supplements in Social Studies, Science and Literature) available from the Department of Education, and books by Sanders<sup>5</sup> and Hedges<sup>6</sup>. Although these works apply to different school subjects, many of the approaches to classification and questioning, and many types of illustrated items are adaptable to Grade IX Mathematics. Further study of the handbook by Bloom will also lead to a deeper insight into principles that the present taxonomy attempts to summarize in such a confined space.

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1. Summary Description of Grade Nine Social Studies Objectives and Test Items (First Edition),
  2. Summary Description of Grade Nine Science Objectives and Test Items (Revised Edition)
  3. Summary Description of Grade Nine Literature Objectives, Test Items and Blueprint
  4. Taxonomy of Language IX Objectives with Illustrative Test Items: A Summary Description.
  5. Sanders, Norris M., Classroom Questions: What Kinds?, New York: Harper & Row, 1966
  6. Hedges, Testing and Evaluation for the Sciences, Belmont, California Wadsworth Publishing Company, Inc., 1966



## PART III

## TOPIC 1: Plane and Space Geometry

1.00 KNOWLEDGE

Knowledge, as defined here, involves the recall of specifics and universals, the recall of methods and processes, or the recall of a pattern, structure or setting.

1.10 Knowledge of Specifics

The recall of specific and isolable bits of information. The emphasis is on symbols with concrete referents.

## 1.11 Knowledge of Terminology

Knowledge of the referents for specific symbols.  
(verbal and non-verbal)

Some Objectives

1. To recall definitions of geometric terms

Illustrative The segments which join corresponding

Item: vertices of the bases of a prism are called

- A. slant heights
- \*B. lateral edges
- C. medians
- D. hypotenuses
- E. cylinders

2. To recall commonly used symbols such as  $\rightarrow$ ,  $\cong$ ,  $\simeq$ ,  $\overline{AB}$ ,  $\angle$ ,  $\triangle$ ,  $\overleftrightarrow{AB}$ , etc.

## 1.12 Knowledge of Specific Facts

Knowledge of what is characteristic of mathematical elements.

Some Objectives

1. To know specific geometric facts

Illustrative The faces of all right prisms are

Item:

- A. regular polygons
- B. quadrilaterals
- C. triangular regions
- D. circular regions
- \*E. polygonal regions

1.20 Knowledge of Ways and Means of Dealing with Specifics

Knowledge of the ways of organizing, studying, judging, and criticizing and knowledge of their existence, not their actual use.

## 1.21 Knowledge of Conventions

Knowledge of characteristic ways of treating and presenting ideas and phenomena.



## Some Objectives

1. To recall the conventions related to geometric symbolism

Ill. Item: If  $\triangle ABC \cong \triangle STR$  which of these is also TRUE?

- A.  $\angle A \cong \angle R$
- B.  $\angle B \cong \angle S$
- C.  $\overline{AB} \cong \overline{RT}$
- D.  $\overline{BC} \cong \overline{ST}$
- \*E.  $\angle A \cong \angle S$

## 1.22 Knowledge of Trends and Sequences

Knowledge of the processes, directions, and movements of phenomena with respect to time.

(Not applicable to Grade IX mathematics)

## 1.23 Knowledge of Classifications and Categories

Knowledge of the classes, sets, divisions, and arrangements which are regarded as fundamental for a given subject field, purpose, argument, or problem.

## Some Objectives

1. To recognize the special classes and categories of geometric figures

Ill. Item: Which of these statements is FALSE?

- A. All rhombuses are parallelograms.
- B. All squares are rectangles.
- C. All squares are rhombuses.
- \*D. All quadrilaterals are parallelograms.
- E. All rectangles are parallelograms.

## 1.24 Knowledge of Criteria

Knowledge of the criteria by which facts, principles, opinions, and conduct are tested or judged.

## Some Objectives

1. To know the rules for classifying geometric figures

Ill. Item: ABCD is a square if

- A. all sides are parallel and congruent.
- B. the opposite sides are parallel and congruent.
- \*C. all sides are congruent and all angles are right angles.
- D. the opposite sides are congruent and all angles are right angles.
- E. the opposite angles are congruent.

## 1.25 Knowledge of Methodology

Knowledge of the methods of inquiry, techniques, and procedures used for problem solving -- not the ability to use these.

Some Objectives

1. To know that an illustrative diagram can simplify a problem
2. To know that formulas can aid in structuring a problem
3. To know the procedure for solving formulas
4. To know the procedure for stating the answer

Ill. Item: Problem: What is the length of a diagonal of a rectangle whose dimensions are 30 ft by 50 ft ?

The following are steps that may be used to solve this problem:

1. Formula
2. Answer
3. Diagram
4. Replacement

The order of these steps is

- A. 1, 4, 2, 3
- B. 1, 4, 3, 2
- C. 3, 1, 2, 4
- D. 1, 2, 3, 4
- \*E. 3, 1, 4, 2

1.30 Knowledge of Universals and Abstractions

Knowledge of the major schemes and patterns by which phenomena and ideas are organized. (Algebraic structures, geometric structures, general mathematical systems)

## 1.31 Knowledge of Principles and Generalizations

Knowledge of particular abstractions which summarize observations of phenomena.

Some Objectives

1. To know specific formulas for finding areas, volumes, perimeters, etc.

Ill. Item: The formula for the lateral area of a right triangular prism is

- A.  $A = \frac{1}{2}bh$
- B.  $A = \frac{1}{3}bh$
- C.  $A = 2\pi rh$
- \*D.  $A = ph$
- E.  $A = ab$

2. To know the principle that  $c/d$  is constant for all circles

Ill. Item: If  $c$  is the circumference and  $d$  is the measure of the diameter of a circle, then  $c/d$

- A. increases as  $\underline{c}$  increases.
- B. increases as  $\underline{d}$  increases.
- \*C. remains constant as  $\underline{c}$  increases.
- D. decreases as  $\underline{c}$  increases.
- E. decreases as  $\underline{d}$  increases.

### 1.32 Knowledge of Theories and Structures

Knowledge of the body of principles and generalizations together with their interrelations.

## 2.00 COMPREHENSION

Comprehension refers to the understanding of what is being communicated and the use of this material or idea being communicated.

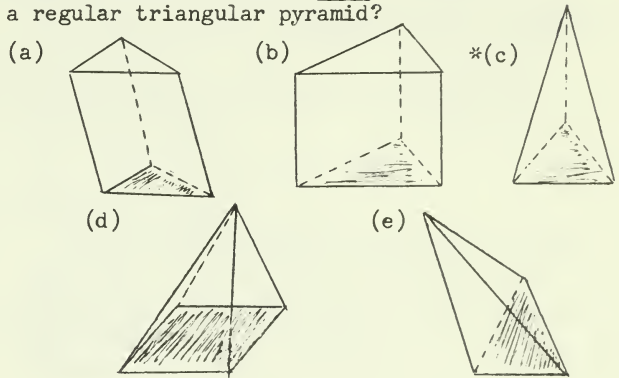
### 2.10 Translation

The ability to give meaning to mathematical statements and to transform them into symbolic form (and conversely).

Some Objectives

1. To translate the information given in a diagram

Ill. Item: Which of these diagrams best illustrates a regular triangular pyramid?



### 2.20 Interpretation

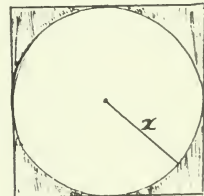
The explanation or summarization of a communication. Interpretation involves a reordering, rearrangement, or a new view of the material.

Some Objectives

1. To express the properties of geometric figures in algebraic terms

Ill. Item: The formula for the area of the shaded portion of this figure is

- A.  $A = 4(x - \pi x)$
- B.  $A = 2x(x - \pi)$
- \*C.  $A = x^2(4 - \pi)$
- D.  $A = x^4(1 - \pi)$
- E.  $A = 4x^2 - \pi$



## 2. To draw conclusions from given data

Ill. Item: Given that (1) in a four-sided polygon the diagonals bisect each other and (2) at least one of the interior angles is a right angle. The polygon may best be described as a

- A. rhombus.
- B. parallelogram.
- \*C. rectangle.
- D. trapezoid.
- E. square.

2.30 Extrapolation

The extension of trends or tendencies beyond the given data to determine implications, consequences, corollaries, effects, etc., which are in accordance with the conditions described in the original communication.

## Some Objectives

1. To predict the effect of altering specific elements in geometric figures

Ill. Item: The volumes of two spheres are in the ratio 8:1. If the radius of the smaller sphere were doubled the volumes would now be in the ratio

- A. 8:2
- \*B. 1:1
- C. 16:1
- D. 8:4
- E. 16:2

### 3.00 APPLICATION

The use of abstractions in particular and concrete situations. The abstractions may be in the form of general ideas, rules of procedures, or generalized methods. The abstractions may also be technical principles, ideas, and theories which must be remembered and applied.

#### Some Objectives

1. To apply mathematical principles to new or practical situations

Ill. Item: The volume of a cubical bin is 343 cu. ft. What would be its capacity if each side were increased by 2 ft?

- A. 8 cu. ft.
- B. 345 cu. ft.
- C. 351 cu. ft.
- D. 686 cu. ft.
- \*E. 729 cu. ft.

Ill. Item: A 10 ft. ladder is leaning against a wall so that it reaches 8 ft. up the wall. If the foot of the ladder were moved 2 ft. further out from the wall, how high up the wall would the ladder reach?

(Students' methods of solving will vary)

4.00 ANALYSIS

The breakdown of a communication into its constituent elements or parts such that the relative hierarchy of ideas is made clear and/or the relations between the ideas are made explicit.

4.10 Analysis of Elements

Identification of the elements included in a communication.

Some Objectives

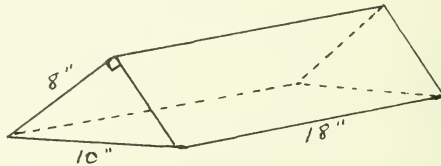
1. To be able to identify relevant and irrelevant data in a problem
  - Ill. Item: A circle has an area of 250 sq. in. If its radius were increased by 5 inches, its circumference would be increased approximately
    - A. 10 in.
    - B. 15.7 in.
    - C. 18.1 in.
    - D. 25 in.
    - \*E. 31 in.
2. To distinguish facts from assumptions

4.20 Analysis of Relationships

The connections and interactions between elements and parts of a communication.

Some Objectives

1. To identify Unspecified relations and to use these to further simplify the material
  - Ill. Item: The lateral surface area of the right prism below is
    - A. 324 sq. in.
    - \*B. 432 sq. in.
    - C. 468 sq. in.
    - D. 480 sq. in.
    - E. 864 sq. in.





## 5.00 SYNTHESIS

The putting together of elements and parts so as to form a whole. This involves the process of working with pieces, parts, elements, etc., and arranging and combining them in such a way as to constitute a pattern or structure not clearly there before.

### 5.10 Production of a Unique Communication

The development of a communication in which the student must convey ideas to others. (Not applicable to Grade IX mathematics)

### 5.20 Production of a Plan or Proposed Set of Operations

The development of a plan of work or the proposal of a plan of operations.

Some Objectives

1. To propose a method of attack for solving an unfamiliar problem

Ill. Item: Suggest a method for locating a point  $O$  in the segment  $NM$  such that  $\angle PON \cong \angle QOM$

$P.$



Ill. Item: A box has a volume of 8 cu. ft. If it is a cube, what is the length of the longest diagonal of the box?

- A. 2 ft
- B. 2.8 ft
- \*C. 3.6 ft
- D. 4 ft
- E. 6 ft

2. To test a hypothesis

Ill. Item: Describe a method for showing that two triangles are similar, using only a compass.

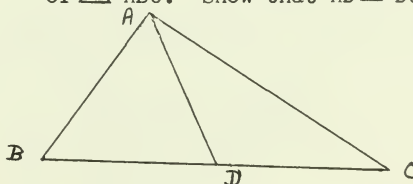
### 5.30 Derivation of a Set of Abstract Relations

The ability to make mathematical discoveries and generalizations

Some Objectives

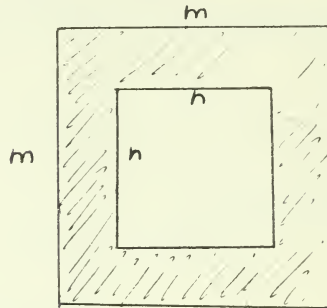
1. To discover the relationship between medians of right triangles and diagonals of rectangles

Ill. Item: Given  $D$  as the midpoint of the hypotenuse of  $\triangle ABC$ . Show that  $AD \cong DC$



2. To find and use the relationship between areas of squares and sides of right triangles

III. Item: Using only a straight edge and compass construct a square equal in area to the shaded figure



## 6.00 EVALUATION

Judgements about the value of material and methods for given purposes. Quantitative and qualitative judgements about the extent to which material and methods satisfy criteria. Use of a standard of appraisal.

### 6.10 Judgements in Terms of Internal Evidence

### 6.20 Judgements in Terms of External Criteria

(This level is too difficult for Grade IX mathematics students and does not lend itself to simple questions within the scope of this knowledge)

## TOPIC 2: Mathematical Systems

1.00 KNOWLEDGE1.10 Knowledge of Specifics

## 1.11 Knowledge of Terminology

## Some Objectives

## 1. To recall certain mathematical terms

Ill. Item: The rule for combining two elements in a set to give a unique element in the same set is

- A. a union.
- \*B. an operation.
- C. numeration.
- D. correspondence.
- E. addition.

## 1.12 Knowledge of Specific Facts

## Some Objectives

## 1. To know what principal roots are irrational numbers

Ill. Item:  $\sqrt{3}$  is a (an)

- A. natural number.
- B. rational number.
- C. integer.
- \*D. irrational number.
- E. counting number.

1.20 Knowledge of Ways and Means of Dealing with Specifics

## 1.21 Knowledge of Conventions

## Some Objectives

## 1. To know meaning of mathematical symbols

Ill. Item: In our mathematics the sign  $-$  is used

- A. to indicate "subtraction" only
- B. to indicate "additive inverse" only
- C. to indicate "negative" only
- \*D. all of the above
- E. none of the above

## 1.23 Knowledge of Classifications and Categories

## Some Objectives

## 1. To know repeating decimals name rational numbers

Ill. Item: .363636 . . . is a (an)

- A. irrational number
- B. terminating decimal
- \*C. rational number
- D. even number
- E. repetend

1.30 Knowledge of Universals and Abstractions

## 1.31 Knowledge of Principles and Generalizations

## Some Objectives

1. To know the properties of mathematical systems

Ill. Item: A property of the real numbers that is illustrated by the statement  $-\sqrt{3} \cdot 7 = 7 \cdot -\sqrt{3}$  is

- A. closure for multiplication
- \*B. commutativity for multiplication
- C. inverse-element for multiplication
- D. associativity for multiplication
- E. square root of real numbers

## 1.32 Knowledge of Theories and Structures

## Some Objectives

1. To know the relationship between the properties of number systems

Ill. Item: A property that the integers, rational numbers, and the real numbers have in common is

- A. closure for subtraction
- B. closure for division
- C. multiplicative inverses
- D. completeness
- E. density

2.00 COMPREHENSION2.10 Translation

## Some Objectives

1. To translate an English statement into symbols

Ill. Item: "The absolute value of a negative real number is the additive inverse of the number."

This statement is best expressed by

- A.  $|x| = -x$
- B.  $|-x| = +x$
- \*C.  $|x| = -x$  if  $x < 0$
- D.  $|x| = x$  if  $x < 0$
- E.  $|-x| = -x$  if  $x > 0$

Ill. Item: The statement "seven is subtracted from the additive inverse of negative five" is best expressed by

- A.  $7 - ^+(-5)$
- B.  $+(-5) \quad 7$
- C.  $-7 - ^-(-5)$
- \*D.  $-(-5) \quad -7$
- E.  $-(5) \quad -7$

\*Most of the remaining categories are not applicable to this content division

## TOPIC 3: Mathematical Logic

1.00 KNOWLEDGE1.10 Knowledge of Specifics

## 1.11 Knowledge of Terminology

Some Objectives

1. To know the meaning of terms used in Mathematical logic

Ill. Item: The antecedent in the sentence "If tomorrow is Tuesday, then today is Monday" is

- A. "if"
- B. "then"
- \*C. "tomorrow is Tuesday"
- D. "today is Monday"
- E. "tomorrow"

## 1.12 Knowledge of Specific Facts

Some Objectives

1. To know symbolism used in mathematical logic

Ill. Item: Suppose A represents "It is sunshining" and B represents "We will play baseball" then  $A \rightarrow B$  represents

- A. "It is sunshining and we will play baseball."
- B. "Only if it is sunshining will we play baseball."
- C. "We will play baseball only if it is sunshining."
- \*D. "If it is sunshining, then we will play baseball."

1.20 Knowledge of Ways and Means of Dealing with Specifics

## 1.21 Knowledge of Conventions

Some Objectives

1. To know the agreements regarding truth values in mathematical logic

Ill. Items: When both statements in a conditional statement are false, the conditional is:

- A. false
- \*B. true
- C. true or false
- D. neither true nor false

## 1.23 Knowledge of Classifications and Categories

## Some Objectives

1. To be able to classify statements in mathematical logic

Ill. Item: "We will go on a trip if and only if the car will start" is a

- A. conditional
- B. conjunction
- \*C. biconditional
- D. disjunction
- E. contrapositive

## 1.24 Knowledge of Criteria

## Some Objectives

1. To know the rules for determining the truth values of a statement

Ill. Item: A conditional is false when the antecedent is

- A. false and the consequent is true
- B. false and the consequent is false
- C. true and the consequent is true
- \*D. true and the consequent is false

## 1.25 Knowledge of Methodology

## Some Objectives

1. To know the method of testing the validity of statements

Ill. Item: To show that  $A \vee B$  and  $\sim B \rightarrow A$  represent equivalent statements for all replacements of A and B, the first step would be to

- A. draw a Venn diagram
- \*B. construct a truth table
- C. substitute statements for A and B
- D. decide if  $A \vee B$  is true
- E. decide if  $\sim B \rightarrow A$  is false

1.30 Knowledge of Universals and Abstractions

## 1.31 Knowledge of Principles and Generalizations

## Some Objectives

1. To know the relationship between the truth values of different kinds of statements

Ill. Item: Which of the following is true?

- \*A. a conditional and its contrapositive have the same truth values
- B. a conditional and its converse have the same truth values
- C. a contrapositive and its converse have the same truth values
- D. a conjunction and a disjunction of the same two statements have the same truth values

2.00 COMPREHENSION2.10 Translation

Some Objectives

1. To be able to translate English sentences into logical symbols

Ill. Item: The statement "If you are a mathematics teacher, then you are intelligent" could be represented by

- A.  $p \text{ or } q$
- B.  $p \text{ and } q$
- C.  $\sim p$
- \*D.  $p \rightarrow q$
- E.  $\sim p \rightarrow q$

2.20 Interpretation

Some Objectives

1. To be able to recognize logically equivalent statements

Ill. Item: The conditional "If the bus will come soon, we will not be late for the game" means the same as

- A. "If we will not be late for the game, then the bus will come soon."
- \*B. "If we will be late for the game, then the bus will not come soon."
- C. "If the bus will not come soon, we will be late for the game."
- D. "If the bus will not come soon, we will not be late for the game."

3.00 APPLICATION

Some Objectives

1. To write logically equivalent statements

Ill. Item: The statement "It is not the case that the doorbell is ringing or the telephone is ringing." is equivalent to

- A. "If the doorbell is not ringing, then the telephone is not ringing."
- \*B. "The doorbell is not ringing and the telephone is not ringing."
- C. "The doorbell is ringing or the telephone is ringing."
- D. "The doorbell is ringing and the telephone is ringing."



4.00 ANALYSIS

## Some Objectives

1. To identify unspecified logical relations and to use these

Ill. Item: If the statement "The team will lose if John is not playing." is false, then the statement "John is playing or the team will not lose" is

- \*A. true
- B. false
- C. cannot tell

## TOPIC 4: Formulas and Variation

1.00 KNOWLEDGE1.10 Knowledge of Specifics

## 1.11 Knowledge of Terminology

Some Objectives

1. To know that letters represent variables

Ill. Item: In the formula  $i = prt$ ,  $p$  is the variable for

- A. profit
- B. per cent
- C. product
- \*D. principal
- E. perimeter

## 1.12 Knowledge of Specific Facts

Some Objectives

1. To know specific formulas

Ill. Item: The formula which indicates the amount of a loan is

- \*A.  $a = p + i$
- B.  $a + p = i$
- C.  $p - i = a$
- D.  $p = br$
- E.  $p = 2\ell + 2w$

1.20 Knowledge of Ways and Means of Dealing with Specifics

## 1.21 Knowledge of Conventions

Some Objectives

1. To recall the conventions related to formulas

Ill. Item: An interest rate of 2% means that the rate is

- A. 2% per month
- B. 2% per  $\frac{1}{4}$  year
- C. 2% per  $\frac{1}{2}$  year
- \*D. 2% per year
- E. 24% per year

## 1.23 Knowledge of Classifications and Categories

Some Objectives

1. To recognize various forms of variation

Ill. Item: The interest on \$ $p$  for 1 year at 5% is expressed by  $i = .05p$ .  $i = .05p$  is an example of  $a(an)$ 

- \*A. direct variation
- B. indirect variation
- C. verification
- D. variation constant
- E. dependent variable

## 1.25 Knowledge of Methodology

## Some Objectives

1. To know the steps in problem solving

Ill. Item: Mr. Jones borrowed \$1200 to pay for a color T.V. He promised to pay the loan a year later with interest at 8%. How much was the payment?

List the steps used in solving this problem.  
(Students' answers may vary)

1.30 Knowledge of Universals and Abstractions

## 1.31 Knowledge of Principles and Generalizations

## Some Objectives

1. To know the various forms of the formula for direct variation

Ill. Item: Which of the following is NOT a direct variation formula?

A.  $\frac{y}{x} = k$

\*B.  $xy = k$

C.  $i = \frac{p}{.06}$

D.  $x = ky$

E.  $y = kx$

2.00 COMPREHENSION2.10 Translation

## Some Objectives

1. To express mathematical information in a formula

Ill. Item: The simple interest for 1 year on a principal of \$p at 6% is 6% of the principal. This situation may be described by

A.  $ik = p$

B.  $\frac{.06}{i} = p$

C.  $i = \frac{p}{.06}$

D.  $ip = k$

\*E.  $i = .06p$

2.20 Interpretation

## Some Objectives

1. To interpret mathematical data and express it as a formula

Ill. Item: The following table shows the relationship between  $i$  and  $p$

$i$	\$7	\$21	\$35	\$70
$p$	\$100	\$300	\$500	\$1000

The relationship between  $i$  and  $p$  is expressed by

- A.  $ip = k$
- B.  $ip = .07$
- \*C.  $\frac{i}{p} = .07$
- D.  $\frac{p}{.07} = i$

2. To know how to rearrange the elements of a formula

Ill. Item: Solve  $p + prt = A$  for  $p$

- A.  $p = A - prt$
- B.  $p = A - rt - 1$
- C.  $p = \frac{A - p}{rt}$
- \*D.  $p = \frac{A}{1 + rt}$
- E.  $p = \frac{A}{1 - rt}$

2.30 Extrapolation

## Some Objectives

1. To predict the effect of altering specific elements of a formula

Ill. Item: The interest formula is  $i = prt$ . If  $r$  remains constant,  $p$  is doubled and  $t$  is tripled, then  $i$  is

- A. doubled
- B. tripled
- C. the same
- \*D. six times as great
- E. four times as great

3.00 APPLICATION

## Some Objectives

1. To apply mathematical principles to new situations

Ill. Item: Mr. Brown wishes to establish an annual scholarship of \$100 by investing a certain amount of money at 5%. How much must be invested?

- A. \$100
- B. \$200
- C. \$1000
- \*D. \$2000
- E. \$20,000

4.00 ANALYSIS

## Some Objectives

1. To be able to critically analyze an offered solution to a problem

Ill. Item: Sam Jones bought a tractor valued at \$2500. He paid \$1000 down and agreed to pay the remainder in 9 months with 8% interest. What was the final payment?  
The following solution was submitted.

Step I - He owed \$1500

Step II - Interest =  $\$1500 \times \frac{8}{100} \times \frac{9}{12} = \$90$

Step III - The final payment was \$2590

Which of the following is true?

- A. All steps are correct
- \*B. I and II are correct
- C. I and III are correct
- D. II and III are correct

5.00 SYNTHESIS

## Some Objectives

1. To propose a plan for solving an unfamiliar problem

Ill. Item: Mr. J. Smith owns a house worth \$10,000 which he rents out for \$125 per month. He pays \$625 taxes and \$175 for repairs on it each year. What rate of interest does he realize on his investment?

- \*A. 7%
- B. 8%
- C.  $8\frac{3}{4}\%$
- D. 15%
- E. 22%

6.00 EVALUATION

## Some Objectives

1. To judge a course of action on the basis of mathematical findings

Ill. Item: Mr. Mercer has a chance to sell his house now for \$14000 and invest the money at 7% OR he can keep the house two years longer and then sell it for \$16000. During the two years he will rent his house for \$200 per month, and he must pay yearly taxes of \$800 and repairs of \$200. Which is the better deal?

- \*A. He should keep the house for 2 years and then sell it.
- B. It will not make any difference what he does.
- C. He should sell the house now and invest the money.

## TOPIC 5a: Algebraic Expressions

1.00 KNOWLEDGE1.10 Knowledge of Specifics

## 1.11 Knowledge of Terminology

Some Objectives

1. To recall specific algebraic terms

Ill. Item: A polynomial contains

- A. at least three monomials
- B. two or more monomials
- C. one or more algebraic expressions
- \*D. one or more monomials
- E. two or more algebraic expressions

## 1.12 Knowledge of Specific Facts

Some Objectives

1. To recall the definition of the "degree" of a polynomial

Ill. Item: A second degree polynomial in one variable contains

- A. two terms
- B. the constant 2
- C. only the first power of the variable and a constant
- D. monomials exactly divisible by 2
- \*E. the second power as the greatest power of the variable

1.20 Knowledge of Ways and Means of Dealing with Specifics

## 1.21 Knowledge of Conventions

Some Objectives

1. To know the order of operations in an expression

Ill. Item: To simplify  $6a - 3 \times 4a + 7$  you would first

- A. subtract 3 from  $6a$
- \*B. multiply 3 and  $4a$
- C. add 7 to  $4a$
- D. add 7 to  $6a$
- E. multiply  $6a - 3$  by  $4a + 7$

## 1.23 Knowledge of Classifications and Categories

Some Objectives

1. To be able to recognize special products

Ill. Item: An expression such as  $4x^6 - 9$  is a (an)

- A. perfect square
- B. trinomial
- \*C. difference of two squares
- D. second degree polynomial
- E. incomplete square



## 1.24 Knowledge of Criteria

## Some Objectives

1. To know the characteristics of algebraic expressions

Ill. Item: An example of a monomial is

- A.  $\sqrt{x}$   
 B.  $\sqrt[2]{x}$   
 \*C.  $2.5x^2y^3$   
 D.  $\frac{x+2}{2}$   
 E.  $3x^{-2}$

## 1.25 Knowledge of Methodology

## Some Objectives

1. To know in what ways algebraic expressions may be factored

Ill. Item: To factor  $6a^2 + 7a$ 

- \*A. remove the greatest common factor  
 B. add the two terms  
 C. complete the square  
 D. divide the expression by 6  
 E. take the square root of the expression

1.30 Knowledge of the Universals and Abstractions in a Field

## 1.31 Knowledge of Principles and Generalizations

## Some Objectives

1. To know the general principle for multiplying two polynomials

Ill. Item: To multiply  $3x$  by  $4x - 6$  we must apply the

- \*A. distributive property of multiplication over addition  
 B. commutative property of addition  
 C. well-defined property of multiplication (multiplication principle)  
 D. associative property of addition  
 E. transitive property of equality

2.00 COMPREHENSION2.10 Translation

## Some Objectives

1. To be able to translate English statements into algebraic expressions

Ill. Item: The square root of the sum of the square of x and the cube of y, written in symbolic form is

A.  $\sqrt{(x^2 + y)^3}$

\*B.  $\sqrt{x^2 + y^3}$

C.  $\sqrt{(x + y^2)^3}$

D.  $\sqrt{2x + 3y}$

## 2.20 Interpretation

Some Objectives

1. To be able to simplify algebraic expressions by re-ordering the elements

Ill. Item:  $(x + 2) + 3x + 4(x - 3) =$

A.  $8x - 1$

\*B.  $8x^2 - 10$

C.  $3x^2 + 10x - 12$

D.  $3x^3 + x^2 - 22x - 24$

E.  $10x^3 - 12$

## 3.00 APPLICATION

Some Objectives

1. To be able to apply mathematical principles in problem solving

Ill. Item: The length of a rectangular lot exceeds its breadth by 20 yds. If each dimension is increased by 20 yds. the area of the lot will be doubled. Find the shorter dimension of the original lot. Show your work below.

A. 20 yds.

B. 30 yds.

C. 35 yds.

\*D. 40 yds.

E. none of these.

## 4.00 ANALYSIS

(Not Applicable)

## 5.00 SYNTHESIS

(Not Applicable)

## 6.00 EVALUATION

(Not Applicable)

## TOPIC 5b: Linearity, Equality, and Inequality

1.00 KNOWLEDGE1.10 Knowledge of Specifics

## 1.12 Knowledge of Specific Facts

Some Objectives

1. To know the laws of exponents

Ill. Item:  $\sqrt{x^2} =$ 

- A.  $2x$
- B.  $|x|$
- \*C.  $x$
- D.  $\frac{1}{2}x^2$
- E.  $\frac{1}{x^2}$

1.20 Knowledge of Ways and Means of Dealing with Specifics

## 1.21 Knowledge of Conventions

Some Objectives

1. To recognize the various mathematical properties as used in solving conditions

Ill. Item: In solving a condition, Joe wrote the following chain of equivalent conditions:

1.  $\frac{-2}{3}x < 6$

2.  $\frac{-3}{2} \left( \frac{-2}{3}x \right) > \frac{-3}{2}(6)$

3.  $\left( \frac{-3}{2} \cdot \frac{-2}{3} \right)x > -9$

4.  $1x > -9$

5.  $x > -9$

Fill in the blanks with the correct property:

In going from 1 to 2 he used the \_\_\_\_\_  
 2 to 3 he used the \_\_\_\_\_  
 3 to 4 he used the \_\_\_\_\_  
 4 to 5 he used the \_\_\_\_\_

## 1.25 Knowledge of Methodology

## Some Objectives

1. To know the procedure for solving an inequality

Ill. Item: If  $3x+17>0$ , then

- \*A.  $x > -17/3$
- B.  $x > -3/17$
- C.  $x > +3/17$
- D.  $x > +17/3$

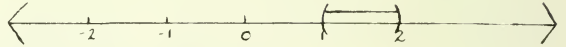
2.00 COMPREHENSION2.10 Translation

## Some Objectives

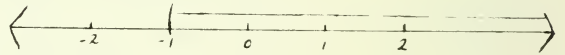
1. To translate a mathematical statement into graphic form

Ill. Item: The graph of the solution set of  $x-1>0 \wedge x-2<0$  is

\*A.



B.



C.



D.



2.20 Interpretation

## Some Objectives

1. To summarize mathematical information in the form of an equation

Ill. Item: Mr. Brown leaves Edmonton at 8:00 a.m. and drives toward Grande Prairie at an average speed of 50 m.p.h. Mr. Jones leaves Edmonton at 8:30 a.m. and proceeds on the same route at an average speed of 60 m.p.h. How long will Mr. Brown be on the road before Mr. Jones overtakes him?

The condition that may be used to represent this problem is

- A.  $50t = 60t - \frac{1}{2}$
- B.  $60t = 50t - 25$
- C.  $t = 60(50 + \frac{1}{2})$
- \*D.  $50t = 60(t - \frac{1}{2})$
- E.  $50t = 60(t + \frac{1}{2})$

3.00 APPLICATION

## Some Objectives

1. To apply algebraic procedures to solve a problem

Ill. Item: The solution set of  $|2x| - 5 \leq 13$  is

- A.  $x < 9$
- B.  $x = 9$
- C.  $-9 < x < 9$
- \*D.  $-9 \leq x \leq 9$

4.00 ANALYSIS

(Not Applicable)

5.00 SYNTHESIS

## Some Objectives

1. To devise an algebraic method for solving a problem

Ill. Item: If you can row upstream to a landing 5 miles away and then row back to your starting point all in 3 hours and 20 minutes, and if the river has an average current of 2 miles per hour, at what speed are you able to row in still water?

(Students' answers will vary)

6.00 EVALUATION

(Not Applicable)

[illegible][illegible]





